Monetary Theory and Policy
TA 2

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Outline

1. **The Effects of Tariffs on Economic Activity**
2. **Identification of the Phillips Curve**
3. **The Forward Guidance Puzzle**

The Effects of Tariffs on Economic Activity

Barnichon and Singh (2025)

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- **Two Identification Strategies:**
  1. **OLS approach:** Exploits that tariff changes do not appear systematically related to the cycle
     - Unique feature: Opposite political parties had opposite views on tariffs
     - This creates quasi-random variation in tariff policy
  2. **IV approach:** Based on narrative dates with long-run motivated tariff changes
- **Trade-offs:** OLS exploits more variance but less robust; IV approach is more robust to remaining endogeneity but exploits smaller variance—we report results from both methods

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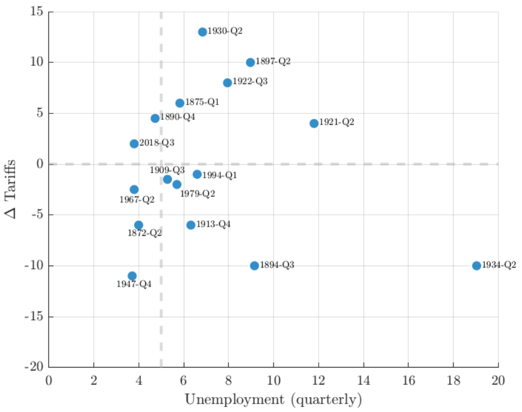
- **Trade-offs:** OLS exploits more variance but less robust; IV approach is more robust to remaining endogeneity but exploits smaller variance—we report results from both methods
- **Key Findings:**
  - Tariffs act as **demand shocks** (rather than supply shocks)
  - Higher tariffs lead to **lower inflation** and **higher unemployment**

On the quasi-randomness of US tariff changes

Figure 3: Tariff changes and unemployment, United States



- Shows **no clear correlation** between tariff changes and the business cycle

Quantifying Quasi-Randomness: Statistical Tests

p-value	Tariff change	Republican victory	Power flip
Major tariff change	.67	.71	.60
1868-2018	—	.20	.31
1868-1939	—	.19	.30
1946-2018	—	.44	.01***

- **Main regression:** $\Delta\tau_i = \alpha + \beta x_i + e_i$ (x_i = unemployment rate)
- **F-test (p-value):** Tests if $\beta \neq 0$ vs. $\beta = 0$. High p-values (> 0.5) $\Rightarrow$ no relationship
- **Result:** No relationship between tariff changes and business cycle

- **Validation:** $d_i = \alpha + \beta X_i + e_i$ where d_i = party dummy or power flip dummy, X_i = unemployment/inflation (contemporaneous + 4 lags)
- **Result:** State of economy does not favor one party over another (p-values > 0.5)
- **Note:** Post-WWII, economic conditions predict power flips, but not tariff changes

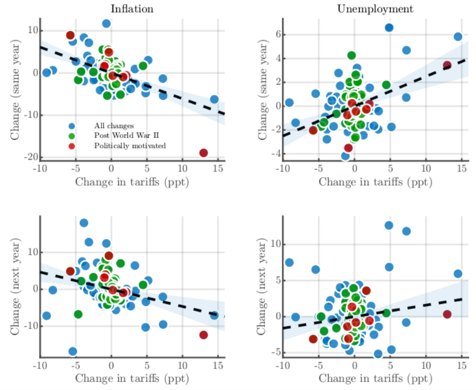
A narrative proxy for tariff shocks

Table 1: Narratively identified tariff shocks, United States (1890–2018)

Year	Policy Change	Tariff	Main Reason for Change
1890	McKinley Tariff Act	Hike	Protecting infant industries
1909	Payne-Aldrich Tariff Act	Cut	Ideological split within Republican party
1922	Fordney–McCumber Tariff Act	Hike	Ideological return to protectionism
1947	Geneva Round	Cut	Free trade preference and promoting post-war stability
1967	Kennedy Round	Cut	Strengthen transatlantic economic integration
1979	Tokyo Round	Cut	Free trade preference
1994	Uruguay Round	Cut	Free trade preference
2018	Trump Tariffs	Hike	National-security and strategic

- Identifies tariff changes that are **exogenous** to economic conditions
- Provides clean variation for the IV approach

Correlation in the Data: Tariffs, Inflation and Economic Activity



- Tariffs change and unemployment are **positively correlated**
- Tariffs change and inflation are **negatively correlated**

The Two Approaches in Practice: OLS Approach

OLS / Recursive VAR

$$y_t = \begin{bmatrix} \Delta\tau_t \\ u_t \\ \pi_t \end{bmatrix}$$

1. Reduced Form VAR (what we estimate):

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + e_t$$

- $e_t = (e_{\Delta\tau,t}, e_{u,t}, e_{\pi,t})'$ are **reduced form residuals** (correlated)
- Estimated by OLS

2. Structural VAR (what we want):

$$B_0 y_t = d + B_1 y_{t-1} + B_2 y_{t-2} + \eta_t$$

The Two Approaches in Practice: OLS Approach

Mapping: Reduced Form $\rightarrow$ Structural VAR

From structural form: $B_0 y_t = d + B_1 y_{t-1} + B_2 y_{t-2} + \eta_t$

Rewriting:

$$y_t = \underbrace{B_0^{-1} d}_{=c} + \underbrace{B_0^{-1} B_1}_{=A_1} y_{t-1} + \underbrace{B_0^{-1} B_2}_{=A_2} y_{t-2} + \underbrace{B_0^{-1} \eta_t}_{=e_t}$$

The mapping: $e_t = B_0^{-1} \eta_t \quad \begin{bmatrix} e_{\Delta\tau,t} \\ e_{u,t} \\ e_{\pi,t} \end{bmatrix} = B_0^{-1} \begin{bmatrix} \eta_{\Delta\tau,t} \\ \eta_{u,t} \\ \eta_{\pi,t} \end{bmatrix}$

The Two Approaches in Practice: OLS Approach

Identification Problem: We need to impose restrictions to identify B_0 from the mapping $e_t = B_0^{-1}\eta_t$

Solution: Cholesky Ordering We impose a lower triangular structure on B_0^{-1}:

Let b_{ij} denote element (i, j) of B_0^{-1}

$$B_0^{-1} = \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Economic Intuition: $\Delta\tau_t$ does not react to contemporaneous u_t or π_t (i.e. due to legislative delay)

Cholesky decomposition: $\Sigma_e = B_0^{-1}\Sigma_\eta(B_0^{-1})' = B_0^{-1}(B_0^{-1})'$ (since $\Sigma_\eta = I$)

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The Two Approaches in Practice: OLS Approach

Recovering Structural Shocks η_t Identification: Cholesky uniquely determines each b_{ij} from Σ_e:

$$b_{11} = \sqrt{\text{Var}(e_{\Delta\tau,t})} \quad b_{21} = \text{Cov}(e_{u,t}, e_{\Delta\tau,t})/b_{11} \quad b_{31} = \text{Cov}(e_{\pi,t}, e_{\Delta\tau,t})/b_{11}$$

$$b_{22} = \sqrt{\text{Var}(e_{u,t}) - b_{21}^2} \quad b_{32} = (\text{Cov}(e_{\pi,t}, e_{u,t}) - b_{31}b_{21})/b_{22} \quad b_{33} = \sqrt{\text{Var}(e_{\pi,t}) - b_{31}^2 - b_{32}^2}$$

Once we have B_0, we can recover structural shocks from reduced form residuals: $\eta_t = B_0 e_t$

"Structural" Tariff Shock: $\eta_{\Delta\tau,t}$ is the first element of $\eta_t = B_0 e_t$

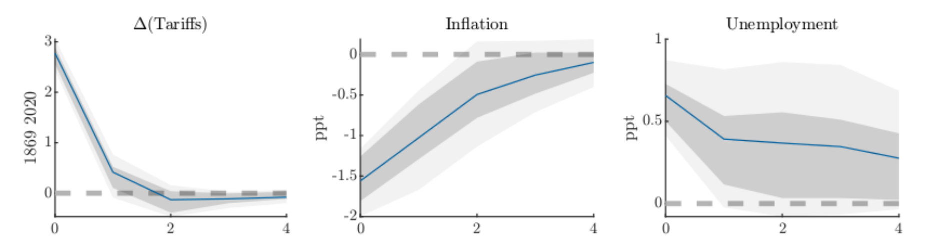
The Two Approaches in Practice: IV Approach

Narrative IV VAR

$$x_t = \begin{bmatrix} \varepsilon_t \\ \Delta\tau_t \\ u_t \\ \pi_t \end{bmatrix}, \qquad x_t = d + B_1 x_{t-1} + B_2 x_{t-2} + e_t$$

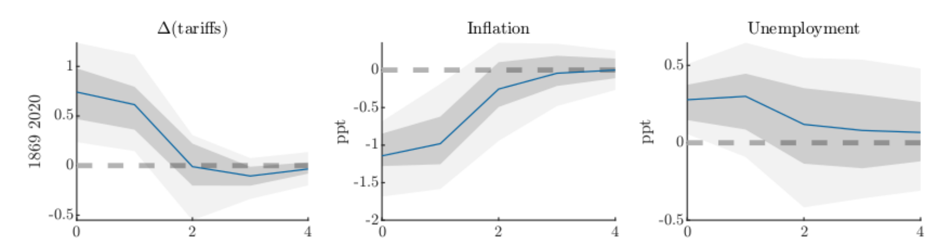
- Yearly VAR(2) in $(\varepsilon_t, \Delta\tau_t, u_t, \pi_t)'$
- ε_t is a **narrative tariff shock series** (nonzero only on "exogenous" tariff dates)
- **Identification Intuition:** ε_t isolates tariff changes plausibly exogenous to the business cycle
- ε_t ordered first; treated as exogenous (no lags of other variables in its equation)
- Identification: Cholesky with ε_t first, i.e. treat ε_t as an external instrument for $\Delta\tau_t$
- **Structural shocks:** $\eta_t = B_0 e_t$ are orthogonal shocks to $(\varepsilon_t, \Delta\tau_t, u_t, \pi_t)'$, with $\eta_{\varepsilon,t}$ the narrative tariff shock

OLS Approach: Impulse Response Functions



- Response to a tariff shock $\eta_{\Delta\tau,t}$ using OLS/Recursive VAR approach
- An increase in tariffs lowers inflation and increases unemployment

IV Approach: Impulse Response Functions



- Response to a narrative tariff shock $\eta_{\Delta\tau,t}$ using IV VAR approach
- Same effects as in the OLS case

Tariff Shocks as Aggregate Demand Shocks?

Standard Theory Prediction (Guerrieri, Lorenzoni and Werning, 2025) Tariff shocks act as **cost-push shocks**

- Higher tariffs ↑ production costs (more expensive imported intermediates)
- Higher tariffs ↑ prices of foreign final goods
- **Prediction:** Higher tariffs → lower economic activity and **higher inflation**

Empirical Finding: Opposite inflation response

- Higher tariffs → higher unemployment and **lower inflation**
- Effects are **akin to adverse aggregate demand shocks**

Key Question: Why do tariff shocks $\eta_{\Delta\tau,t}$ behave like demand shocks rather than supply shocks?

Mechanisms: Uncertainty and Wealth Channels

Possible Explanations:

1. Uncertainty Channel: (Leduc and Liu, 2016)

- Tariff shock $\eta_{\Delta\tau,t}$ creates (or coincides with) uncertain economic environment
- Uncertainty $\downarrow$ consumers' and investors' confidence
- $\rightarrow$ Depresses economic activity and puts downward pressure on inflation

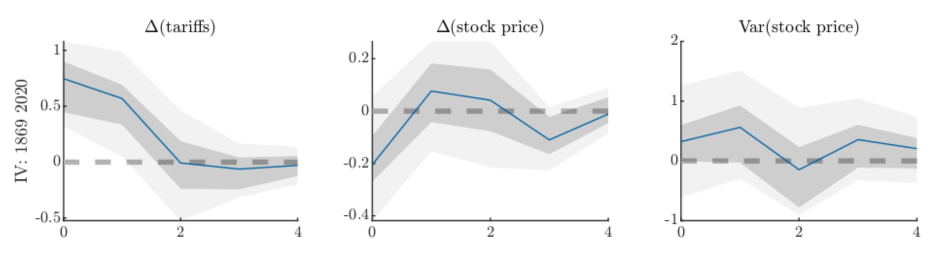
2. Wealth Channel:

- Adverse tariff shock $\eta_{\Delta\tau,t} \rightarrow$ drop in asset prices
- $\rightarrow$ Depresses aggregate demand
- $\rightarrow$ Higher unemployment and lower inflation

Testing the Mechanisms: Extend VAR with:

- Stock price index (common stock prices from NBER Macrohistory, 1871+)
- Stock market volatility (annual volatility from variance of monthly price changes)

Mechanism: How Tariffs Affect the Economy



- Impulse responses estimated from IV-approach VAR with $(\varepsilon, \Delta\tau, \pi, u, i, \Delta(SP), \mathrm{Var}(SP))$
- Both the **wealth** and **uncertainty** channels affect the economy
- A tariff increase lowers stock market valuation and raises stock price volatility

Summary: Barnichon and Singh (2025)

Main Finding: Tariffs act as **aggregate demand shocks**, not supply shocks

Key Results:

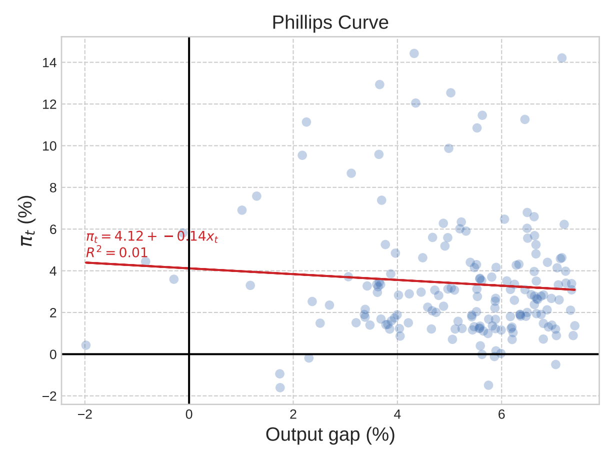
- Higher tariffs $\rightarrow$ **lower inflation** and **higher unemployment**
- Effects are consistent across both OLS and IV identification strategies
- Mechanism: Tariffs operate through **uncertainty** and **wealth** channels
  - Uncertainty: Tariff shocks create economic uncertainty, depressing confidence
  - Wealth: Tariff shocks reduce asset prices, depressing aggregate demand

The Identification of the Phillips Curve

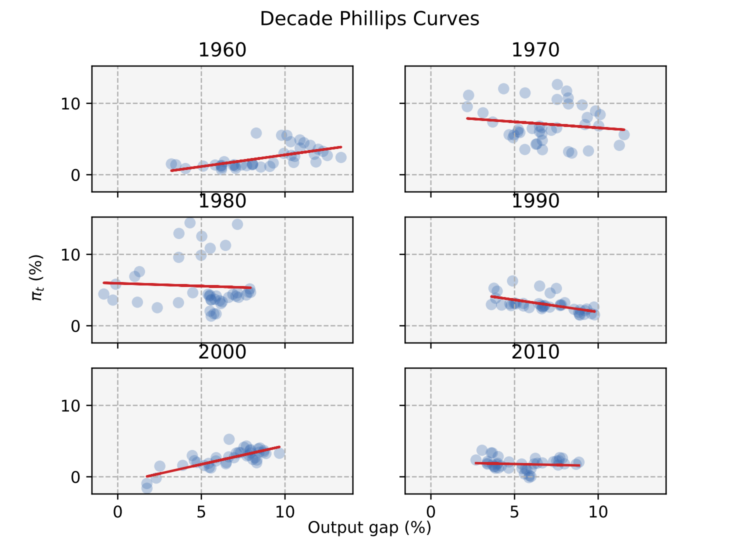
Does the Phillips Curve exist? (i)

NK Phillips Curve: $\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \lambda x_t + \nu_t$

OLS regression: $\pi_t = \gamma_1 + \gamma_2 x_t + \varepsilon_t$



Does the Phillips Curve exist? (ii)



The Issue

- Clearly, the Phillips curve is **well alive** in the NK model:

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- Today, we shall see why a simple OLS regression does not work
- Material here is roughly based on McLeay & Tenreyro (2020)

A role for optimal MP

- Start from a model in which the NKPC is **by assumption** there

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- Solving the NKPC forward upon plugging the MPR, we get Computations

$$\pi_t = \frac{\omega}{\lambda^2 + \omega(1 - \beta\rho)} \nu_t$$

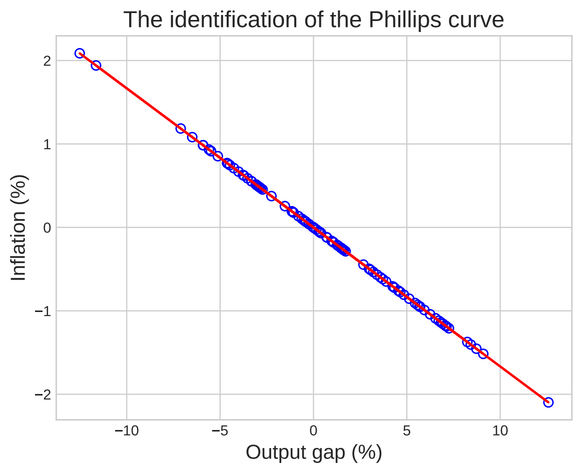
which we can use to simulate data assuming some values for the parameters (details in McLeay & Tenreyro (2020)) and the following exogeneous process for ν_t:

$$\nu_t = \rho \nu_{t-1} + \eta_t \sim \mathcal{N}(0, \sigma_\eta)$$

- $\Rightarrow$ simulate ν_t say 100 times and compute $\pi_t = \frac{\omega}{\lambda^2+\omega(1-\beta\rho)}\nu_t$ and $x_t = -\frac{\lambda}{\omega}\pi_t$

The Identification Issue

OLS regression: $\pi_t = \gamma_1 + \gamma_2 x_t + \varepsilon_t$



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  - But then, since $v_{t+1} = \rho v_t + \eta_t$,
$$\varepsilon_t = v_t + \beta \mathbb{E}_t[\pi_{t+1}] = v_t + \beta \mathbb{E}_t\left[\frac{\omega}{\lambda^2+\omega(1-\beta\rho)} v_{t+1}\right] = \left(1 + \frac{\beta\rho\omega}{\lambda^2+\omega(1-\beta\rho)}\right) v_t$$

    so: $\mathbb{E}[x_t \varepsilon_t] \neq 0$

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- We saw that the problem in estimating $\pi_t = \gamma_1 + \gamma_2 x_t + \varepsilon_t$ is $\mathbb{E}[x_t \varepsilon_t] \neq 0$

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- From OLS,

$$
\begin{aligned}
\mathbb{E}[\hat{\gamma}_2] &= \frac{\mathrm{Cov}(\pi_t, x_t)}{\mathrm{Var}(x_t)} = \\
&= \frac{\mathrm{Cov}(\gamma_1 + \gamma_2 x_t + \varepsilon_t, x_t)}{\mathrm{Var}(x_t)} = \\
&= \frac{\mathrm{Cov}(\gamma_1, x_t) + \mathrm{Cov}(\gamma_2 x_t, x_t) + \mathrm{Cov}(\varepsilon_t, x_t)}{\mathrm{Var}(x_t)} = \\
&= 0 + \gamma_2 \frac{\mathrm{Cov}(x_t, x_t)}{\mathrm{Var}(x_t)} + \frac{\mathrm{Cov}\left[\left(1 + \frac{\beta\rho\omega}{\lambda^2 + \omega(1-\beta\rho)}\right)\nu_t, -\frac{\lambda}{\lambda^2 + \omega(1-\beta\rho)}\nu_t\right]}{\mathrm{Var}\left(-\frac{\lambda}{\lambda^2 + \omega(1-\beta\rho)}\nu_t\right)} = \\
&= \gamma_2 - \underbrace{\frac{\lambda^2 + \omega(1-\beta\rho) + \beta\rho\omega}{\lambda}}_{\text{Bias}}
\end{aligned}
$$

because $\mathrm{Cov}(a\nu_t, b\nu_t) = ab\mathrm{Cov}(\nu_t, \nu_t) = ab\mathrm{Var}(\nu_t)$, and $\mathrm{Var}(a\nu_t) = a^2\mathrm{Var}(\nu_t)$

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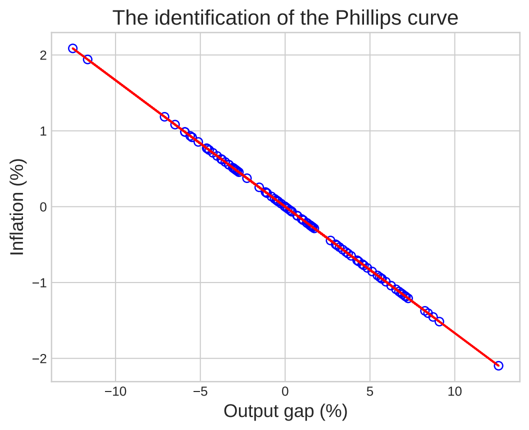
The Identification Issue revisited

OLS regression: $\pi_t = \gamma_1 + \gamma_2 x_t + \varepsilon_t$



$$\Rightarrow \hat{\gamma}_2 = -\frac{\omega}{\lambda}(< 0)$$

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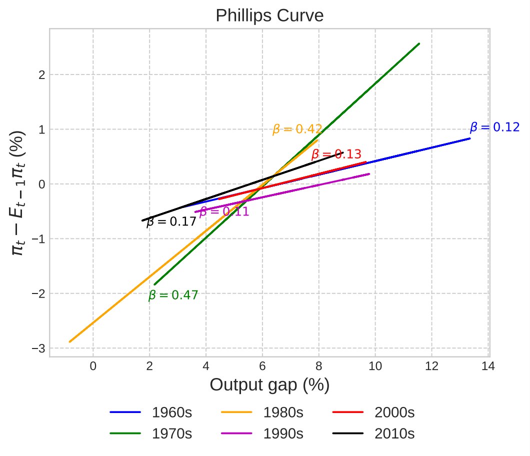
  is that $\mathbb{E}_t[x_t \varepsilon_t] \neq 0$
- Controlling for $\mathbb{E}_t[\pi_{t+1}]$ lowers the endogeneity concern, however **still an issue** because of simultaneity between x_t and ν_t

Controlling for Expectations?

OLS Regression: $\pi_t = \gamma_1 + \gamma_2 x_t + \varepsilon_t$

Expectations: $\varepsilon_t = \pi_t^e + \nu_t$



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- **Regional Phillips Curves**
  - In region c, x_{ct} reacts to regional shocks ξ_{ct} and the aggregate shock ν_t
  - Using a panel, time FE control for ν_t since it is common across regions c
  - The residual variation in x_{ct} depends only on ξ_{ct}

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- In the latter two, two competing mechanisms: deflationary pressure due to demand shortage, and inflationary pressure due to higher marginal costs

The Forward Guidance Puzzle

The Simple NK Model

You are given the three-equations representation of the NK model:

$$x_t = \mathbb{E}_t[x_{t+1}] - \sigma(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) \tag{NKIS}$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \lambda x_t \tag{NKPC}$$

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- Term $\varepsilon_{t,t-j(t)}$ is a forward guidance shock
- $\varepsilon_{t,t-j(t)}$ shock to short term real interest rate that becomes known in period $t - j(t)$.

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Lemma (Law of Iterated Expectations)
Let $\{\mathcal{I}_t\}_{t\geq 0}$ be such that $\mathcal{I}_{t+j} \subseteq \mathcal{I}_{t+k}$ for all $j \leq k$. Then,

$$\mathbb{E}_{t+j}[\cdot] = \mathbb{E}_{t+j}[\mathbb{E}_{t+k}[\cdot]]$$

for all $j \leq k$. Note that this implies that $\mathbb{E}_t[\cdot] = \mathbb{E}_t[\mathbb{E}_{t+i}[\cdot]]$ for all i.

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&= \cdots = \lim_{T\to\infty} \mathbb{E}_t\left[x_{t+T}\right] - \sigma\mathbb{E}_t\left[\sum_{k=0}^{\infty} i_{t+k} - \pi_{t+k+1} - r_{t+k}^n\right]
\end{aligned}$$

Some unplesant algebra: IS

- Recall the NK IS:

$$x_t = \mathbb{E}_t[x_{t+1}] - \sigma(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) \quad \text{(NKIS)}$$

- We have (recall $\mathbb{E}_t[\mathbb{E}_{t+k}[\cdot]] = \mathbb{E}_t[\cdot]$ for all $k \geq 0$):

$$\begin{aligned}
x_t &= \mathbb{E}_t[\mathbb{E}_{t+1}[x_{t+2}]] - \sigma\mathbb{E}_t[i_{t+1} - \mathbb{E}_{t+1}[\pi_{t+2}] - r_{t+1}^n] - \sigma(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) = \\
&= \mathbb{E}_t[x_{t+2}] - \sigma\mathbb{E}_t\left[\sum_{k=0}^{1} i_{t+k} - \pi_{t+k+1} - r_{t+k}^n\right] = \\
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&= \cdots = \underbrace{\lim_{T\to\infty} \mathbb{E}_t\left[x_{t+T}\right]}_{=0 \text{ By Assumption}} - \sigma\mathbb{E}_t\left[\sum_{k=0}^{\infty} i_{t+k} - \pi_{t+k+1} - r_{t+k}^n\right] \\
&= -\sigma\mathbb{E}_t\left[\sum_{k=0}^{\infty} i_{t+k} - \pi_{t+k+1} - r_{t+k}^n\right]
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Some unpleasant algebra: Phillips Curve

- Let's work out the Phillips curve:

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- We got the following:

$$x_t = -\sigma \mathbb{E}_t \left[\sum_{k=0}^{\infty} i_{t+k} - \pi_{t+k+1} - r_{t+k}^n \right] \tag{NKIS}$$

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$$= -\sigma \sum_{k=0}^{\infty} \mathbb{E}_t \left[\varepsilon_{t+k,t+k-j(t+k)} \right]$$

Conventional MP

- We need the two previous equations:

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- From (NKIS), we have

$$x_t = -\sigma \left\{ \mathbb{E}_t[\varepsilon_{t,t}] + \sum_{k=1}^{\infty} \mathbb{E}_t \left[\varepsilon_{t+k,t+k-j(t+k)}\right] \right\} = \sigma$$

$$\forall i \geq 1, \; x_{t+i} = -\sigma \left\{ \sum_{k=0}^{\infty} \mathbb{E}_{t+i} \left[\varepsilon_{t+i+k,t+i+k-j(t+i+k)}\right] \right\} = 0$$

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- Hence, expansionary conventional MP pushes output and inflation up

Forward Guidance

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- From (NKIS), focus on shocks $\varepsilon_{t+k,t}$ (all others are 0):

$$x_{t+i} = -\sigma \sum_{k=0}^{\infty} \mathbb{E}_{t+i}[\varepsilon_{t+i+k,t}] = \begin{cases} \sigma & \text{if } i \leq T-t \\ 0 & \text{else} \end{cases}$$

since there is no discounting

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  since there is no discounting
- Then, from the (NKPC):

$$\pi_t = \lambda \sum_{k=0}^{T} \beta^k \sigma + \lambda \sum_{k=T+1}^{\infty} \beta^k \times 0 = \lambda\sigma \frac{1-\beta^T}{1-\beta}$$

A Puzzle rises

$$\pi_t = \lambda\sigma\frac{1-\beta^T}{1-\beta}$$

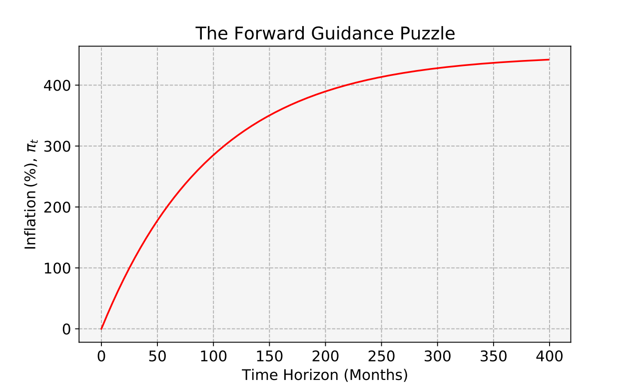
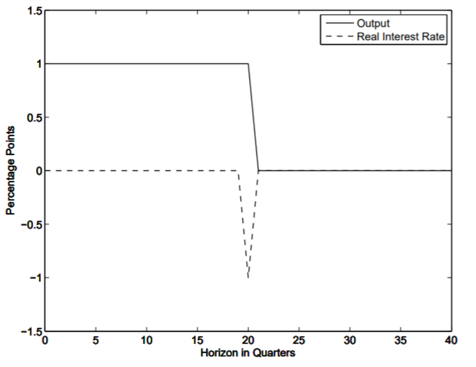


Figure 1: Inflation response to a forward guidance shock $\varepsilon_{t+T,t} = -1$, for varying T.

Understanding the puzzle

- Consider the shock of the exercise:
  - Today, the CB announces that it will cut *i* in 20 qrtrs
  - The nominal, and hence real, rates only change in 20 years from now
  - Output, however, jumps up **immediately** and adjusts in 20 qrtrs



Understanding the puzzle

WHY?

- Put yourself in the shoes of the household
- For her, the shock changes the relative cost of consumption between periods T and $T + 1$ **only**
- So, consumption growth can only change in period $T \Rightarrow$ consumption is a step function
- Inflation, per (NKPC), reflect the cumulated change in consumption

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- Inflation, per (NKPC), reflect the cumulated change in consumption

Elephant(s) in the room: Euler equation & Complete markets

- EE determines the step nature of consumption
- But perfect consumption smoothing is achieved **iff** the household can borrow at will

Fixing the NK bug: McKay et al (2016)

- **How**: keep EE, but abandon complete markets

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  - People wish to maintain a buffer wealth in case it rains
  - To take full advantage of a FG shock, the household would run down on her assets
  - That's because she would need to borrow until consumption is cheap, ie until the shock materializes
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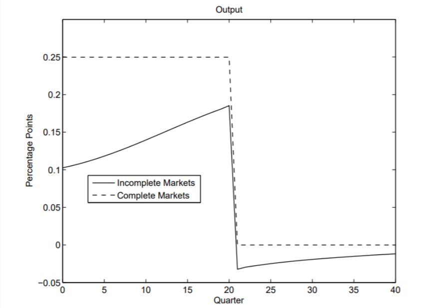
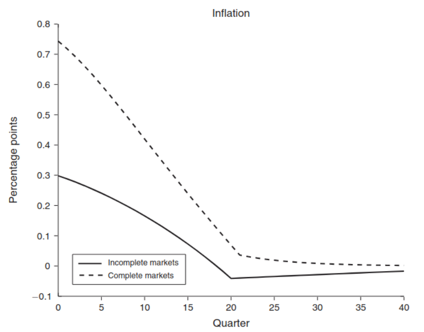
- Huge (-ly complicated, recent) literature on heterogeneity in macro GE models [Short Review] – very active field of research

Fixing the NK bug: McKay et al (2016)

Discounted IS: $x_t = \alpha \mathbb{E}_t[x_{t+1}] - \sigma(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n)$





- **Output:** Initial response ∼40% of complete markets; falls below steady state after rate change
- **Inflation:** Initial response ∼40% of complete markets

Appendix

Derivations of forward NKPC

- Start from $\pi_t = -\omega/\lambda x_t$, from which $x_t = -\lambda/\omega \pi_t$
- Since $\nu_t = \rho\nu_{t-1} + \eta_t$, it is $\mathbb{E}_t[\nu_{t+1}] = \rho\nu_t$
- Plugging the first expression into the NKPC we have

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} - \frac{\lambda^2}{\omega}\pi_t + \nu_t$$

- Solving for π_t and iterating forward,

$$\begin{aligned}\pi_t &= \frac{\omega}{\omega+\lambda^2}\left(\beta\mathbb{E}_t\pi_{t+1} + \nu_t\right) = \\ &= \frac{\omega}{\omega+\lambda^2}\left\{\beta\mathbb{E}_t\left[\frac{\omega}{\omega+\lambda^2}\left(\beta\mathbb{E}_{t+1}\pi_{t+2} + \nu_{t+1}\right)\right] + \nu_t\right\} = \\ &= \left(\frac{\omega}{\omega+\lambda^2}\right)^2\beta^2\mathbb{E}_t\pi_{t+2} + \left(\frac{\omega}{\omega+\lambda^2}\right)\nu_t + \left(\frac{\omega}{\omega+\lambda^2}\right)^2\beta\rho\nu_t = \\ &= \cdots = \left(\frac{\omega}{\omega+\lambda^2}\right)\sum_{k=0}^{\infty}\left(\frac{\beta\rho\omega}{\omega+\lambda^2}\right)^k\nu_t\end{aligned}$$

- Solving for the summation we get

$$\pi_t = \nu_t\frac{\omega}{\omega+\lambda^2}\frac{1}{1-\frac{\beta\rho\omega}{\omega+\lambda^2}} = \nu_t\frac{\omega}{\lambda^2+\omega(1-\beta\rho)}$$

Back